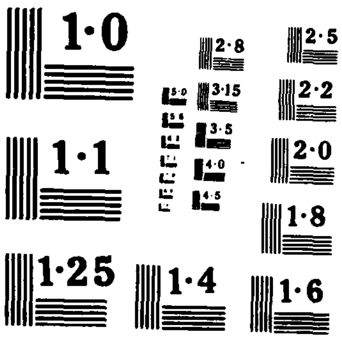
AD-A169 548   A BRIEF REVIEW OF ADAPTIVE NULL STEERING TECHNIQUES(U)   1/1
ROYAL SIGNALS AND RADAR ESTABLISHMENT MALVERN (ENGLAND)
J G MCWHIRTER FEB 86 RSRE-MEMO-3939 DRIC-BR-99244

UNCLASSIFIED   F/G 20/4   NL


1·0
2·8
2·5
3·15
2·2
3·5
2·0
4·0
1·1
4·5
1·8
1·25
1·4
1·6

UNLIMITED BR99244 (2)

AD-A169 548

RSRE
MEMORANDUM No. 3939

ROYAL SIGNALS & RADAR ESTABLISHMENT

A BRIEF REVIEW OF ADAPTIVE NULL
STEERING TECHNIQUES

Author: J G McWhirter

PROCUREMENT EXECUTIVE,
MINISTRY OF DEFENCE,
RSRE MALVERN,
WORCS.

RSRE MEMORANDUM No. 3939

UNLIMITED 86 7 15 102

ROYAL SIGNALS AND RADAR ESTABLISHMENT

Memorandum 3939

Title: A BRIEF REVIEW OF ADAPTIVE NULL STEERING TECHNIQUES

Author: J G McWhirter

Date: February 1986

SUMMARY

A brief theoretical review of adaptive null steering is presented. The basic theory is first outlined in the context of sidelobe cancellation systems as well as general antenna arrays. Various approaches to the practical implementation of adaptive null steering are then discussed. These fall into the two main categories of closed loop methods and direct solution methods. The closed loop methods are very cost-effective and suitable in principle, for either analogue or digitial processing. However their rate of convergence is fundamentally limited and too slow for some applications. The direct solution methods do not suffer from this problem but tend to be suitable only for digital processing and are more expensive from the computational point of view. However they are well suited to parallel processing and now provide a very practical alternative due to recent advances in VLSI circuit technology. A brief discussion on the effects of multi-path propagation on adaptive null steering systems concludes this ~~brief~~ reviews.

A BRIEF REVIEW OF ADAPTIVE NULL STEERING TECHNIQUES

J G McWhirter

PREFACE

This brief review was prepared originally as one section of a TTCP report on "Antenna Array Signal Processing" produced by a study group from the KTP3 panel. It only provides a theoretical discussion of the key signal processing techniques and how they are perceived to be evolving. It does not address any of the wider system or technology related issues and was never intended as a comprehensive review of the entire subject. It has been produced here as a stand alone document in response to a request from several readers who felt that it provides a good introduction to the subject and should be more readily available. I am grateful to them for their comments and hope that this memorandum may indeed prove to be useful in that context.

A BRIEF REVIEW OF ADAPTIVE NULL STEERING TECHNIQUES

J G McWhirter

CONTENTS

1 INTRODUCTION

The ability to control the phase and amplitude of the received signal in each channel of an antenna array makes it possible to implement various types of adaptive signal processing. In adaptive antenna array signal processing the vector of complex weights $\underline{w}$ and hence the shape of the received beam is determined as a function of the received signal data and can change in response to the signal environment. The most common application of adaptive beamforming is adaptive null steering which constitutes a powerful technique for the suppression of jammer signals in a radar or communications systems.

2 BASIC THEORY

In this section the problem of adaptive null steering will be considered in its most general form without reference to specific implementation or application.

2.1 Sidelobe Cancellation

The technique of adaptive null steering was pioneered in the early 1960's by P Howells (then with General Electric in Syracuse) and S Applebaum (Syracuse University Research Corporation) in the context of (coherent) side-lobe cancellation (SLC)[1]. A typical sidelobe cancellation system is depicted in figure 1. It consists of a main, high gain antenna whose output is designated as channel P+1 and P auxiliary antennae. The auxiliary antenna gains are designed to approximate the average sidelobe level of the main antenna gain pattern. The amount of desired target signal received by the auxiliaries is assumed to be negligible compared to the target signal d in the main channel. The purpose of the auxiliaries is to provide independent replicas of jamming signals in the sidelobes of the main pattern for cancellation. The auxiliary outputs are weighted and summed and the combined signal is added to the signal in the primary channel. The problem is to find a suitable means of controlling the weight $\underline{w}$ so that the maximum possible cancellation is achieved.

In the case of SLC it can easily be shown that the maximum output SNR is obtained when the total output power is minimized provided that the target signal d in the main antenna is not correlated with any of the auxiliary signals. The weight vector $\underline{w}$ must therefore be chosen to minimize the quantity

$$E = \langle |e|^2 \rangle \qquad (1)$$

where

$$e = y + \underline{x}^T \underline{w} \qquad (2)$$

and y and x_i ($i = 1,2 \ldots P$) denote the complex amplitudes of the signals in

the main and auxiliary antennae respectively. It follows that the optimum weight vector is given by the well-known Wiener-Hopf equation

$$\underline{M}\underline{w}^{*} = -\underline{\rho} \qquad (3)$$

where

$$\underline{M} = \langle \underline{x}\underline{x}^{+} \rangle \qquad (4)$$

is the P x P covariance matrix of the auxiliary channel signals

and

$$\underline{\rho} = \langle \underline{x}y^{*} \rangle = \langle \underline{x}d^{*} \rangle \qquad (5)$$

is the vector of cross-correlations between the auxiliary channel signals and the signal from the primary antenna. A detailed knowledge of the array geometry (which determines the relative phase and amplitude of each signal) is not essential for the purposes of SLC unless it is desired to interpret the weight vector in terms of an overall antenna response pattern.

2.2 General Arrays

The techniques of adaptive null steering can also be applied to the more general antenna array configuration depicted in figure 2. It comprises P individual antenna elements whose outputs are multiplied by complex weighting factors and summed together to give a single combined output. Each of the P channels contains an interference component whose complex amplitude is denoted by $x_k^{(I)}$. The envelope power in the k^{th} channel is denoted by $m_{kk}^{(I)}$ and the covariance of $x_k^{(I)}$ and $x_l^{(I)}$ by

$$m_{k\ell}^{(I)} = \langle x_k^{(I)} x_\ell^{*(I)} \rangle \qquad (6)$$

The target signal, when it occurs is assumed to be present in the channels in proportion to the known complex values s_k ie the signal in the k^{th} channel is given by αs_k where denotes the level and time variation of the signal. The vector

$$\underline{s} = \begin{bmatrix} s_1 \\ s_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ s_P \end{bmatrix}$$

constitutes a generalized 'source' or 'direction' vector which describes the (relative) phase and amplitude at each receiving element of a given wavefront arising from the target. A detailed knowledge of the array geometry is essential in order to define this vector. Eg in a linearly spaced array with equally spaced elements the vector $\underline{s}$ corresponding to a

far field source at a direction θ from mechanical boresight is given by

$$s_k = \exp\left(j\frac{2\pi k\delta}{\lambda}\sin\theta\right) \tag{7}$$

where δ is the element spacing and λ is the wavelength. This is the situation depicted in figure 2.

The problem again is to choose the complex weight vector $\underline{w}$ so that the SNR at the output of the combiner is maximized. The signal and interference outputs are given respectively by

$$e_s = \alpha\underline{s}^T\underline{w} \tag{8}$$

and

$$e_I = \underline{x}^{(I)T}\underline{w} \tag{9}$$

where $\underline{x}^{(I)} = \{ x_i^{(I)};\ i=1,2 \ldots P \}$ is the vector of received interference components. The signal output power is therefore given by

$$E_S = \langle|e_S|^2\rangle = \langle|\alpha|^2\rangle\underline{w}^T\underline{s}\ \underline{s}^+\underline{w}^* \tag{10}$$

where no assumptions about coherent detection have been made. The expected noise output power takes the form

$$E_I = \langle|e_I|^2\rangle = \underline{w}^T\underline{M}^{(I)}\underline{w}^* \tag{11}$$

where

$$\underline{M}^{(I)} = \langle\underline{x}^{(I)}\underline{x}^{+(I)}\rangle = \{m_{k\ell}^{(I)}\} \tag{12}$$

is the P x P covariance matrix of the noise components. It is not difficult to show that the output SNR attains the maximum value

$$E_S/E_I = \langle|\alpha|^2\rangle\ \underline{s}^+\underline{M}^{-1}\underline{s} \tag{13}$$

when the weight vector $\underline{w}$ is given by the equation

$$\underline{M}^{(I)}\underline{w}^* = \mu\underline{s} \tag{14}$$

where μ is an arbitrary gain constant.

It is not possible in every application to measure the interference components alone. However maximizing the output SNR is equivalent to maximizing the ratio of the output signal power to the total output power (signal + interference) provided that the interference and signal waveforms are uncorrelated. Denoting the total received signal by

$$\underline{x} = \underline{x}^{(I)} + \alpha\underline{s} \tag{15}$$

the total output power is given by

$$E_{S+I} = \langle |e_S + e_I|^2 \rangle = \underline{w}^T \underline{M} \underline{w}^* \tag{16}$$

where

$$\underline{M} = \langle \underline{x}\ \underline{x}^+ \rangle$$

$$= \underline{M}^{(I)} + \langle |\alpha|^2 \rangle\ \underline{s}\ \underline{s}^+ \tag{17}$$

ie the P x P covariance matrix of the output signal plus noise from the P channels. The ratio E_S/E_{S+I} is maximized when the weight vector satisfies the equation

$$\underline{Mw}^* = \mu \underline{s} \tag{18}$$

It can easily be shown that the solution to equation (18) must also satisfy equation (14) and so the two are analytically equivalent. However they have different numerical properties which must be taken into account when considering any practical implementation. The more general solution in equation (18) will be assumed in the remainder of this report.

Determining the optimum weight vector $\underline{w}$ for a general adaptive antenna array can also be formulated as a constrained minimization problem defined as follows. Find the weight vector $\underline{w}$ which minimizes the total output power as defined in equation (1) but with

$$e = \underline{x}^T \underline{w} \tag{19}$$

subject to the constraint that

$$\underline{s}^T \underline{w} = \beta \tag{20}$$

This constraint ensures that the antenna gain in the target 'look-direction' is held at a constant value, and hence the signal power in equation (8) is maintained during adaptation. The constrained minimization problem defined above can be solved quite readily using the conventional method of Lagrange undetermined multipliers. The solution is given by

$$\underline{Mw}^* = \beta^* \underline{s} / (\underline{s}^+\ \underline{M}^{-1}\ \underline{s}) \tag{21}$$

which is clearly equivalent to that given in equation (18), the arbitrary gain constant μ being chosen to ensure that the constraint in equation (20) is satisfied.

It is interesting to note that the results derived for adaptive SLC may be derived as a special case of the more general antenna array results above. Since the target signal in SLC is present only in the P+1th channel the appropriate source vector is

$$\underline{t}^T = [0,0,0.........0,1] \tag{22}$$

and the optimum P+1 element weight vector $\underline{w}'$ must satisfy

$$\underline{M}'\underline{w}'^* = \mu\underline{t} \tag{23}$$

where $\underline{M}'$ is the (P+1) x (P+1) covariance matrix of all the channels. Equation (23) may be partitioned in the form

$$\begin{bmatrix} \underline{M} & \underline{\rho} \\ \underline{\rho}^+ & E_{p+1} \end{bmatrix} \begin{bmatrix} \underline{w}^* \\ w^*_{p+1} \end{bmatrix} = \begin{bmatrix} \underline{o} \\ \mu \end{bmatrix} \tag{24}$$

where E_{P+1} is the power output of the main channel and $\underline{M}$ and $\underline{\rho}$ are defined in equations (4) and (3). It follows that the weight vector for the auxiliary channels must satisfy

$$\underline{M}\underline{w}^* = -w^*_{P+1}\underline{\rho} \tag{25}$$

which is equivalent to the result in equation (3) since the parameter μ and hence the weight w_{P+1} may be chosen arbitrarily.

So far in this section it has been assumed that all signal and noise components are narrowband and a single complex quantity has been used to describe the output from each channel. However, each complex signal sample may be replaced by two real samples as illustrated in figure 3. The sample time for the second of these is delayed relative to the first by 1/4f where f is the appropriate centre frequency and a separate real weight factor w_i is applied to each one.[3] Alternatively the baseband Iand Q signals may be treated as two independent real signals. This approach has significant advantages for adaptive nulling since the algorithm is released fromunnecessary constraints imposed by the complex signal representation and becomes less sensitive to problems of I and Q imbalance. The theory outlined above is clearly applicable to the 2P real signals which must be combined in this situation. It can also be applied to situations in which the received signals are broadband.[3] The signals may then be represented by means of a sequence of delayed sample values which could be stored in tapped delay lines as illustrated in figure 4. For a signal bandwidth B, the appropriate delay interval is 1/4B and the number of delays is given by B/f_{res} where f_{res} is the required frequency resolution. Each sample in every receiver channel is multiplied by an individual (real) weighting factor w_i and the resulting products are summed together to form the combined output. For the purpose of applying the theory outlined above, the PL samples (assuming P antenna elements and L samples in each delay lines) are simply treated as elements

of a single sample vector $\underline{x}$ and the output of the combiner may again be expressed in the form $\underline{w}^T\underline{x}$ where $\underline{w}$ is the corresponding PLx1 weight vector. In the case of SLC the output of the combiner is added to the signal received by the primary antenna and the weights are adjusted to ensure that the output power is minimized. In the general antenna array case the weights are adjusted to minimize the output power of the combiner itself subject to L constraints of the form

$$\underline{s}_i^T\underline{w} = \beta_i \qquad i = 1,2,\ldots\ldots L \tag{26}$$

which can be used to ensure that the desired frequency response in a given look direction is maintained.[4]

Most of the methods described in this section can be applied either in element space or in beam space although the former will be assumed in this report. Element space refers to the situation outlined in figure 2 where the adaptive nulling is performed by linearly combining the signals which emerge from the antenna elements. Beam space refers to the situation in which a preliminary fixed linear combination of the signals is performed to produce P beams with some desired characteristics as illustrated in figure 5. Since a linear combination of the beams also constitutes a linear combination of the received signals, adaptive nulling in beam space should yield the same overall antenna response as adaptive nulling in element space even though a different weight vector is computed. The same theory can be applied in each situation and no distinction will therefore be drawn between the two types of application.

3 IMPLEMENTATION

In this section we will review the main techniques which have been devised for implementing adaptive null steering including some very recent developments. The methods may be divided into two main categories. These are (1) Closed loop or feedback control techniques and (2) Direct solution methods (often referred to as 'open-loop'). Broadly speaking closed loop methods are cheaper and simpler to implement than direct solution methods. By virtue of their self correcting nature, they do not require components which have a wide dynamic or high degree of linearity and so they are well suited to analogue implementation. However closed loop methods suffer from the fundamental limitation that their speed of response must be restricted in order to achieve stable operation. Direct solution methods on the other hand do not suffer from problems of slow convergence but in general they require components of such high accuracy and wide dynamic range that they can only be realised by digital means. Of course closed loop methods can also be implemented using digital circuitry, in which case the constraints on numerical accuracy are greatly relaxed and the total number of arithmetic operations is much reduced by comparison with direct solution methods.

3.1 Closed Loop Methods

In this subsection we will briefly review the main closed loop methods both analogue and digital. These methods are now well established and were discussed in a previous report.

3.1.1 Howells-Applebaum

Much of the pioneering work on adaptive antenna arrays was carried out by Howells and Applebaum in the 1960's.[1] The analogue correlation loop which they developed is illustrated schematically in figure 6 for a simple two channel configuration. The auxiliary signal x is multiplied by a complex weight factor w and added to the signal in the primary channel y. The weight is derived by correlating x with the residual signal e. The correlation process is either carried out at RF using suitable mixers or at IF using an analogue multiplier and lowpass filter (or integrator). When it has converged to a steady state the output of the loop is uncorrelated with the signal x provided that the gain of the amplifier is sufficiently high. This type of canceller has developed considerably over the last two decades and is now capable of achieving a level of performance in the laboratory comfortably in excess of that likely to be required in the field. A fairly cheap version implemented as a thin film hybrid circuit has been produced by the Hughes Aircraft Company.[2]

Figure 7 shows schematically how a number of correlation cancellation loops may be used in parallel to implement a multiple sidelobe cancellation system. When each correlation loop has converged to a steady state the residual signal e is uncorrelated with each of the auxiliary signals x_k and so the energy of the residual signal is minimized. It is not difficult to show that the weight vector is then given by the expression

$$(\underline{M} + \underline{I}_p/G)\underline{w}^* = -\underline{\rho} \qquad (27)$$

where $\underline{I}_p$ is the PxP identity matrix and $\underline{M}$ and $\underline{\rho}$ are as defined previously. This is equivalent to the Wiener-Hopf solution given in equation (3) provided that the amplifier gain G is large.

Figure 8 illustrates the use of several parallel correlation cancellation loops to perform adaptive nulling in the case of a general antenna array for which the desired look direction is specified by the vector $\underline{s}$. The weights w_k are derived by correlating each signal x_k with the output signal e, adding the correlation output to the desired vector component s_k and then using a high gain amplifier. In this case the steady state weight vector is given by the equation

$$(\underline{M} + \underline{I}_p/G)\underline{w}^* = \underline{s} \qquad (28)$$

which clearly takes the same form as the optimum control law given in equation (18) provided, once again, that the amplifier gain G is sufficiently large.

3.1.2 Widrow LMS Algorithm

The Least Mean Square (LMS) algorithm developed by Widrow is an entirely digital, closed loop control algorithm suitable for adaptive nulling.[3] Figure 9 illustrates the use of this algorithm for multiple sidelobe cancellation. The P signals $x_k(n)$ are multiplied by complex weighting coefficients w_k and summed together to produce the secondary signal $\sigma(n)$ which, in turn, is added to the primary signal $y(n)$ to produce the output residual signal

$$e(n) = \underline{x}^T(n)\underline{w} + y(n) \tag{29}$$

The vector of weights $\underline{w}$ is updated according to the formula

$$\underline{w}(n+1) = \underline{w}(n) + 2\mu e^*(n)\underline{x}(n) \tag{30}$$

where $\underline{x}(n)$ denotes the vector of signal samples which enter the combiner at the n^{th} discrete time sample and μ is a constant gain factor. The algorithm is particularly efficient, requiring only $\sim$4P real multiplications and $\sim$4P real additions per sample time to update the weight vector. The update formula for each coefficient w_k is clearly of the form

$$w_k(n+1) = w_k(n) + 2\mu e^*(n)x_k(n) \tag{31}$$

which defines a simple digital correlation cancellation loop of the type illustrated in figure 10. The LMS algorithm, in effect, constitutes P such loops operating in parallel with a common gain factor and is the digital equivalent of the Howells-Applebaum technique discussed above.

Widrow has shown that if the weight vector is updated according to equation (30) then

$$\langle \underline{w}^*(n) \rangle \xrightarrow[n\to\infty]{} -\underline{M}^{-1}\underline{\rho} \tag{32}$$

(ie the weight vector tends in the mean to the optimum value given in equation (3)) provided that the gain constant μ satisfies the condition

$$0 < \mu < \frac{1}{\lambda_{max}} \tag{33}$$

where λ_{max} denotes the largest eigenvalue of the covariance matrix $\underline{M}$.

Footnote: For simplicity it has been assumed that one complex multiplication is equivalent to 4 real multiplications and 2 real additions.

The rate of convergence depends on the value of μ. For small values of μ the algorithm converges slowly and the weight vector does not fluctuate too much about the mean. For large values of μ the algorithm converges more rapidly but the weight vector is subject to larger fluctuations due to the fact that the integration time of the loop and hence the statistical accuracy is reduced. These fluctuations in turn lead to 'misadjustment noise' which causes the output energy to increase above its optimum level. The time constant of adaption for the k^{th} normal component of the weight vector (ie the component associated with the k^{th} eigenvector of $\underline{M}$) is given by

$$\tau_k = \frac{1}{2\mu\lambda_k} \tag{34}$$

where λ_k is the k^{th} eigenvalue and so, if the spread of eigenvalues is large, the smaller components must suffer very slow convergence if the stability condition (33) is to be satisfied. This fundamental limit to the overall rate of convergence is common to all of the closed loop techniques for adaptive nulling.

In the above discussion it has been assumed for consistency that the P signals $x_k(n)$ which enter the combiner are complex. However it is worth pointing out that the LMS algorithm may also be applied to the 2P real signal components from a P-element narrow-band array of the type depicted in figure 3 or the PL real signals from a broadband receiver array of the type illustrated in figure 4.[3]

So far the Widrow LMS algorithm has been discussed only in the context of adaptive sidelobe cancellation. It can also be applied to the problem of adaptive nulling for a general antenna array but most of the methods which have been proposed for incorporating the look direction constraint vector are rather cumbersome. Widrow suggested the use of a pilot signal to emulate the effect of the primary signal in a sidelobe canceller.[3] The scheme which he proposed is shown in figure 11. The pilot signal d is fed into each receiver channel with the relative phase and amplitude factors appropriate to a signal received by the antenna array from the required look direction, ie the signal vector $d\underline{s}$ is input to the array. The LMS algorithm is then used to minimise the sum of the array output and the desired signal. As a result the array is constrained to have a strong response in the required look direction whilst nulling out other uncorrelated signals. The resulting weight vector must also be applied to the received signals in the abscence of the pilot signal (using an additional 'slave processor') in order to determine the time output of the array. This technique is expensive to implement and has been shown to lead in general to a biassed solution.

Griffiths[5] has suggested a different technique for applying the Widrow LMS algorithm to adaptive nulling with a general array. He assumes that the cross-correlation vector between the desired signal d(n) and the vector of received signals $\underline{x}(n)$ is known. The update equation (30) for the weight vector is expressed in the form

$$\underline{w}(n+1) = \underline{w}(n) + 2\mu(y^*(n) + \sigma^*(n))\underline{x}(n) \qquad (35)$$

and the quantity $y^*(n)\underline{x}(n)$ is replaced by its mean value

$$\underline{\rho} = \langle y^*(n)\underline{x}(n)\rangle = \langle d^*(n)\underline{x}(n)\rangle \qquad (36)$$

to provide the alternative update formula

$$\underline{w}(n+1) = \underline{w}(n) + 2\mu(\underline{\rho} + \sigma^*(n)\underline{x}(n)) \qquad (37)$$

This equation involves a mixture of average and simultaneous quantities but, as with the conventional LMS algorithm the weight vector $\underline{w}(n)$ converges in the mean to the optimum value given in equation (3). The weight vector update in this case requires ~6P real multiplications and ~6P real additions per sample time.

If the update formula in equation (37) is modified to take the form

$$\underline{w}(n+1) = \underline{w}(n) + 2\mu(-\underline{s} + \sigma^*(n)\underline{x}(n)) \qquad (38)$$

where $\underline{s}$ is an appropriate look-direction constraint vector then the weight vector should again converge in the mean according to equation (37) but with $\underline{\rho}$ replaced by $-\underline{s}$ ie

$$\langle \underline{w}^*(n)\rangle \xrightarrow[n\to\infty]{} M^{-1}\underline{s} \qquad (39)$$

This is of course the optimum control law of equation (18). In effect equation (38) is the digital equivalent of the technique proposed by Applebaum for applying a directional constraint with analogue closed loops as illustrated in figure 8. It does not require any knowledge about the form of the desired signal d(n) - only its direction of arrival.

Frost proposed another method for applying the Widrow LMS algorithm to adaptive nulling with a general array. He included the look-direction constraint (20) by introducing a Lagrange undetermined multiplier into the gradient descent algorithm. The modified algorithm takes the form

$$\underline{w}(n+1) = \underline{F}[\underline{w}(n) + 2\mu e^*(n)\underline{x}(n)] + \underline{f} \qquad (40)$$

with $\underline{w}(o) = \underline{f}$

where $\underline{f}$ is a P-element vector given by

$$\underline{f} = \underline{s}(\underline{s}^T\underline{s})^{-1}\beta \qquad (41)$$

and $\underline{F}$ is the PxP projection matrix given by

$$\underline{F} = \underline{I} - \underline{s}(\underline{s}^T\underline{s})^{-1}\underline{s}^T \qquad (42)$$

The update formula (40) is clearly much more complicated to implement than the basic LMS algorithm and requires an additional 8P real multiplications and 10P real additions per sample time. However it does operate in such a way as to ensure that the constraint equation (20) is always satisfied exactly. This is not the case with the techniques which Widrow and Griffiths proposed.

Frost's method was originally suggested for use with L simultaneous linear constraints in a broadband application.[4] However we have chosen to describe it here for a single constraint in order to facilitate comparison with the previous techniques. In the more general formulations the constraint vector $\underline{s}$ becomes a PLxL constraint matrix and the scalar β becomes an L-element vector but the formula is otherwise identical.

A much simpler technique for incorporating the constraint equation (20) exactly has recently been proposed by McWhirter.[6] It was originally developed for use with a direct solution algorithm based on the method of QR decomposition which is discussed in section 3.2.4. However it is worth describing at this stage since the technique is equally applicable to the Widrow LMS algorithm. The technique can be explained quite simply as follows. The constraint in equation (20) may be written in the form

$$w_p = \beta - \hat{\underline{s}}^T \hat{\underline{w}} \qquad (43)$$

where $\hat{\underline{s}}$ and $\hat{\underline{w}}$ denote the first P-1 elements of the vectors $\underline{s}$ and $\underline{w}$ respectively and it has been assumed without loss of generality that $s_p = 1$. By substituting this expression for w_p into equation (19) it is possible to express the residual signal e in the form

$$e = \underline{Z}^T\hat{\underline{W}} + \beta x_p \qquad (44)$$

where

$$\underline{z} = \underline{x} - x_p\hat{\underline{s}} \qquad (45)$$

$\langle|e|^2\rangle$ must then be minimized with respect to the unconstrained P-1 element weight vector $\hat{\underline{w}}$. Equation (44) clearly takes the same form as equation (2) which describes the side-lobe cancellation problem. The term βx_p plays the role of the primary signal y while the transformed vector $\underline{z}$ corresponds to the auxiliary signal vector $\underline{x}$. The general array problem with a single linear constraint has been transformed quite simply into an equivalent "end-element-clamped" configuration identical to that for SLC. The "end-element-clamped" configuration may therefore be considered as the canonical form for adaptive null steering and it is assumed in the remainder of this report that any technique which is suitable for SLC is equally applicable to the general array problem by applying this transformation technique.

The transformation may be applied using the type of pre-processor which is illustrated in Figure 12. It is implemented using only ~4P real multiples and ~4p real additions. In effect the p^{th} element (all elements are assumed to be equivalent in the general array problem) is arbitrary chosen as the primary channel which will in general receive signals from the required look direction. The signal in this channel is then subtracted from the signal in each of the other channels using an appropriate phase and amplitude weighting which ensures that the resultant signal has a null in the required look direction. Consequently, adaptive combination of the resultant "auxiliary" signals cannot cancel any look direction signal in the primary channel.

3.1.3 Hybrid Analogue/Digital Techniques

So far in this section we have described closed loop processing methods which are either entirely analogue or entirely digital. However a number of hybrid analogue/digital methods have also been proposed in which the weight computation is carried out digitally but the weighting itself is performed by analogue means. This combines the wide bandwidth and signal handling capability of analogue circuits with the flexibility and high dynamic range of digital circuitry. Figure 13 illustrates schematically an adaptive sidelobe canceller based on a hybrid loop which has been produced by the Hughes Aircraft Company.[7] In this case the multiplication of the residual signal by each of the auxilliary signals is also carried out using an analogue multiplier and the product is passed through a losw-pass filter before being digitized. The weight vector can then be computed in a flexible way using, for example, techniques such as Powell's conjugate gradient method for accelerated convergence. If the low pass filter were omitted, the weight vector could be computed using the LMS algorithm given in equation (30) or even Frost's constrained LMS algorithm given in equation (40) (assuming a general array with no main beam signal). The hybrid approach in general allows the weights to be applied at a much higher frequency than they are computed, the A/D conversion being carried out at less than the Nyquist sample rate of the signals. This important advantage of the hybrid analogue/digital approach is not just relevant to closed loop techniques but also to the direct solution methods which are discussed in the next section. In any situation where the weights are derived explicitly it should be assumed that they may be applied by either analogue or digital means and so the use of hybrid analogue/digital techniques will not be discussed further in this chapter.

3.2 Director Solution Methods

Very fast adaptive nulling is considered to be essential in a number of important military applications. For example the antenna array may be mounted on a high speed rotating platform, it may have to cope with rapidly switched jammers or it may be required to change frequency rapidly for the purposes of spread spectrum communications. In such situations direct solution methods are essential and a number of these will now be discussed.

3.2.1 Sample Matrix Inversion

Sample Matrix Inversion (SMI) which was first proposed by Brennan, Reed and Mallett[8] is probably the best known and most obvious direct solution method. It simply involves forming an estimate of the covariance matrix $\underline{M}$ from samples of the signals received by each element of the array and solving equation (3) or equation (21) directly to obtain the corresponding weight vector. The covariance matrix estimate takes the form

$$\underline{M} = \underline{X}^T(n)\underline{X}^*(n) \tag{46}$$

where

$$\underline{X}(n) = \begin{bmatrix} \underline{x}^T(1) \\ \underline{x}^T(2) \\ \cdot \\ \cdot \\ \underline{x}^T(n) \end{bmatrix} \tag{47}$$

is the matrix of received data samples and, as before, $\underline{x}(n)$ denotes the vector of signal samples received across the array at the n^{th} sample time. Each element of M is simply given by

$$m_{ik} = \sum_{j=1}^{n} x_i(j)x_k^*(j) \tag{48}$$

When equation (46) is used to estimate the covariance matrix M, equation (3) corresponds exactly to the Gauss normal equations for linear least squares estimation. The solution $\underline{w}$ is therefore the vector of weights which minimizes the sum of the squares of the output residuals

$$E(n) = \{|e(1)|^2 + |e(2)|^2 \;.....+ |e(n)|^2\}^{\frac{1}{2}} \tag{49}$$

where e(n) is defined in equation (29). In matrix notation $\underline{w}$ is the vector of weights which minimizes $\|\underline{e}(n)\|$ where

$$\underline{e}(n) = \underline{X}(n)\underline{w} + \underline{y}(n) \tag{50}$$

and we have defined

$$\underline{e}(n) = \begin{bmatrix} e(1) \\ e(2) \\ e(3) \\ \cdot \\ \cdot \\ \cdot \\ e(n) \end{bmatrix} \quad (51) \qquad \text{and} \qquad \underline{y}(n) = \begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ \cdot \\ \cdot \\ \cdot \\ y(n) \end{bmatrix} \quad (52)$$

Similarly, equation (21) defines the optimum least squares solution to the problem of minimizing $\| \underline{e}(n) \|$ where

$$\underline{e}(n) = \underline{X}(n)\underline{W} \qquad (53)$$

subject to the constraint in equation (20).

The SMI technique requires $\sim 2nP^2$ real multiplications and $\sim 2nP^2$ real additions in order to form the covariance matrix. In addition, $\sim 4P^3/3$ real multiplications and $\sim 4P^3/3$ real additions are required to compute the weight vector solution (by Gaussian elimination). It should be noted that when $n \gg P$, computation of the covariance matrix is the dominant task, requiring $\sim 2P^2$ real multiplications and $\sim 2P^2$ real additions per sample time. Brennan, Reed and Mallett showed that for a P-element array only $\sim 2P$ data vectors are required when forming the sample covariance matrix in order to achieve an improvement in the output signal to noise power ratio which is within 3 dB of the optimum statistical value. The rate of adaptation is much faster therefore than that of a closed loop algorithm. It is governed only by the statistical accuracy required and not by factors such as stability and convergence time.

It is worth pointing our at this stage that "soft constraints" can also be incorporated within the SMI technique. A soft constraint corresponding to equation (20) is introduced by treating the constraint vector $\underline{s}$ as though it were another data snapshot and appending it to the data matrix $\underline{X}(n)$. The constant β is treated as the corresponding value for the signal in the primary channel. Several constraints may be incorporated in this way if required. Such constraints are not satisfied exactly, the importance of each one being comparable only to that of a single data snapshot within the overall least squares minimization. The influence of any soft constraint may of course be enhanced by multiplying the corresponding equation by a suitable scaling factor. As the scaling factor is increased, the influcence of a soft constraint approaches that of the equivalent hard constraint. However this technique is not recommended for handling hard constraints since it must inevitably lead to a more ill-conditioned matrix and so increase the dynamic range requirements of the processor.

Soft constraints may be introduced in this way to any of the direct solution algorithms discussed in section 3.2. The same comments and conclusions apply and so the subject will not be discussed further in this report.

The sample matrix inversion technique leads quite naturally to a processor architecture of the type illustrated schematically in figure 14 (for SLC). It is quite conventional and comprises a number of distinct components - one to form and store the covariance matrix estimate, one to compute the solution of equation (3) and one to apply the resulting weight vector to the auxiliary signal data. This data must be stored in a suitable memory while the weight vector is being computed. The system also requires a number of high speed data communication buses and a sophisticated control unit to deliver the appropriate sequence of instructions to each component.

This type of architecture is obviously complicated, extremely difficult to design and not very suitable for VLSI.

Not only does the direct solution of equation (3) lead to a complicated circuit architecture, it is also very poor from the numerical point of view. The problem of solving a system of linear equations like those defined in equation (3) can be ill-conditioned and hence numerically unstable. The degree to which a system of linear equations is ill-conditioned is determined by the condition number of the coefficient matrix. The condition number of a matrix $\underline{X}$ is defined by

$$C(\underline{X}) = |\lambda_1/\lambda_N| \qquad (54)$$

where λ_1 and λ_N are the largest and smallest (non-zero) singular values respectively.[9] The larger $C(\underline{X})$ the more ill-conditioned is the system of equations. It follows from equation (46) that

$$C(\underline{M}(n)) = C(\underline{X}^T(n)\ \underline{X}^*(n)) = C^2(\underline{X}(n)) \qquad (55)$$

and so the condition number of the estimated covariance matrix $\underline{M}(n)$ is much greater than than of the corresponding data matrix $\underline{X}(n)$. Any numerical algorithm which avoids forming the estimated covariance matrix explicitly and operates directly on the data is likely to be much better conditioned.

3.2.2 Gram-Schmidt

One such algorithm which is very popular in the current adaptive antenna literature,[10] is based on the modified Gram-Schmidt orthogonalization procedure and operates as follows. Define the n x (P+1) matrix

$$\underline{\phi}(n) = [\underline{\phi}_1,\ \underline{\phi}_2,\ \ldots\ \underline{\phi}_{P+1}] = [\underline{X}(n),\ \underline{y}(n)] \quad (56)$$

The first column of $\underline{\phi}(n)$ (ie the vector of all data entering the first channel of the combiner up to time t_n) is taken as the first vector $\underline{q}_1$ of a new orthogonal set. The remaining vectors $\underline{\phi}_i$ ($i = 2,3 \ldots P+1$) are then made orthogonal to $\underline{q}_1$ by applying the simple projection formula

$$\underline{\phi}_i' = \underline{\phi}_i - \underline{q}_1^* \left(\frac{\underline{q}_1^* \cdot \underline{\phi}_i}{|\underline{q}_1|^2}\right) \qquad (57)$$

The vector $\underline{\phi}_2'$ is taken as the second member $\underline{q}_2$ of the new orthogonal set and this in turn is made orthogonal to the remaining vectors $\underline{\phi}_i'$ ($i = 3,4 \ldots p+1$) by applying the projection formula

$$\underline{\phi}_i'' = \underline{\phi}_i' - \underline{q}_2^* \left(\frac{\underline{q}_2^* \cdot \underline{\phi}_i'}{|\underline{q}_2|^2}\right) \qquad (58)$$

Note that since $\underline{q}_1$ is orthogonal to $\underline{q}_2$ and to $\underline{\phi}_i'$ it must also be orthogonal to $\underline{\phi}_i''$. The vector $\underline{\phi}_3''$ is then taken as the third member $\underline{q}_3$ of

the orthogonal set and the process is continued until a complete set of P+1 orthogonal vectors is obtained. Now the column vector $\underline{q}_{p+1}$ has been constructed by adding to the vector $\underline{y}(n)$ a linear combination of the columns in the matrix $\underline{X}(n)$. It must therefore be of the form

$$\underline{q}_{p+1} = X(n)\underline{w} + \underline{y}(n) \tag{59}$$

which is identical to that of the general residual vector $\underline{e}(n)$ defined in equation (50). Furthermore, since $\underline{q}_{p+1}$ is orthogonal to the other vectors $\underline{q}_1 \dots \underline{q}_p$ and hence to the columns of $\underline{X}(n)$ it must in fact be the residual vector for which $E(n) = \|\underline{e}(n)\|$ is minimized. It therefore provides the required output from the adaptive linear combiner.

In total the Modified Gram-Schmidt algorithm requires $\sim 4nP^2$ real multiplications and $\sim 4nP^2$ real additions ie $\sim 4P^2$ real multiplications and $\sim 4P^2$ real additions per sample time. The corresponding hardware requirements and achievable throughout rates were discussed by Dillard in a previous KTP3 report[11]

The modified Gram-Schmidt algorithm can also be applied to the general adaptive antenna problem either by making use of the constraint preprocessor technique described in section (3.1.2) or by solving the Wiener-Hopf equation for the transformed data matrix

$$\underline{Q}(n) = [\underline{q}_1, \underline{q}_2, \underline{q}_3 \cdots\cdots \underline{q}_p] \tag{60}$$

Denoting the transformation from $\underline{X}(n)$ to $\underline{Q}(n)$ by

$$\underline{X}(n)\underline{T}(n) = \underline{Q}(n) \tag{61}$$

where $\underline{T}(n)$ is an upper triangular matrix, the constrained minimization problem defined by equations (53) and (20) may be expressed as follows. Minimize $\| \underline{e}(n) \|$ where

$$\underline{e}(n) = \underline{Q}(n)\ \underline{w}' \tag{62}$$

subject to the constraint that

$$\underline{s}'^T \underline{w}' = \beta \tag{63}$$

where

$$\underline{s}'^T = \underline{s}^T \underline{T}(n) \tag{64}$$

The optimum weight vector $\underline{w}'$ is then given by the corresponding Wiener-Hopf equation

$$\underline{D}(n)\ \underline{w}'^* = \frac{\beta^*\ \underline{s}'}{\underline{s}'^+ \underline{D}^{-1}(n) \underline{s}'} \tag{65}$$

where

$$\underline{D}(n) = \underline{Q}^T(n)\underline{Q}^*(n) \qquad (66)$$

Since $\underline{Q}(n)$ is orthogonal, $\underline{D}(n)$ is a simple diagonal matrix and so equation (65) may be solved very easily. This approach was discussed during our visit to the Hughes aircraft company who would seem to favour the Modified Gram-Schmidt algorithm for many of their applications.

Of the two methods, the pre-processor technique should be more reliable since the constraint is satisfied exactly before the adaptive weighting is carried out and there is no need to solve for the weight vector explicitly. However, in situations where it is necessary to repeat the adaptive process for a number of different look directions, the pre-processor method is computationally expensive because the entire Gram-Schmidt procedure must be repeated for each one. Solving equation (65) for each look direction is obviously more efficient for such applications.

The modified Gram-Schmidt algorithm described above is known to have good numerical properties.[12] However it does not lead to a particularly suitable circuit architecture. This is due to the fact that the orthogonalization procedure is carried out column by column and so the entire data matrix $\underline{X}(n)$ must be obtained before the operation can commence. This of course leads to a considerable overhead in the amount of memory and control circuitry which is required. It also means that as each new row of data is received, the entire procedure must be repeated in order to update the least-squares estimate. As a result the modified Gram-Schmidt tends to be used on one block of data after another. It may be implemented using the type of triangular archtecture illustrated in Figure 15. On each cycle of the process each cell inputs two complete column vectors _ and $\underline{q}$ and produces the output vector $\underline{\phi}'$ where

$$\underline{\phi}' = \underline{\phi} - \underline{q}^* \left(\frac{\underline{q}^* \cdot \underline{\phi}}{|\underline{q}|^2}\right) \qquad (67)$$

An algorithm which operates row by row and can be implemented recursively would be much more suitable from the point of view of circuit architecture. For this reason the modified Gram-Schmidt algorithm is often computed approximately using a technique which will be referred to as sequential decorrelation.[13]

3.2.3 Sequential Decorrelation

The sequential decorrelation process may also be carried out using a triangular array of processors as illustrated in figure 15. In this case each processor is a simple digital decorrelator which receives two correlated input sequences $q(i)$ and $\phi(i)$ $(i = 1,2 \ldots n)$. On each clock cycle it inputs the current values $\phi(i)$ and $q(i)$ and produces the output value

$$\phi'(i) = \phi(i) - q^*(i)\,\frac{V(i)}{U(i)} \qquad (68)$$

where

$$V(i) = V(i-1) + q^*(i)\phi(i) \qquad (69)$$

and

$$U(i) = U(i-1) + |q(i)|^2 \qquad (70)$$

In this way each cell produces the output sequence

$$\phi'(i) = \phi(i) - q^*(i) \frac{\sum_{j=1}^{i} q^*(j)\phi(j)}{\sum_{j=1}^{i} |q(j)|^2} \qquad (71)$$

which is (asymptotically) uncorrelated with the input sequence q(i). The output value $\phi'(i)$ is passed southwards to the next cell on the next clock cycle. From equation (71) it can be seen that in the sequential decorrelation process the value of $\phi'(i)$ is computed before the complete sequence of data q(i) and $\Phi(i)$ (i = 1,2 ... n) have been received and before calculation of the correlation coefficient at time t_n has been completed. As a result the output sequences $\Phi'(i)$ and q(i) (i = 1 ... n) are not truly orthogonalized as in the proper modified Gram-Schmidt algorithm and the sequential decorrelation process is numerically inferior. Particular case is need, for example, to avoid the problem which arises when the denominator in equation (68) is identically zero during initial stages of the calculation. It is worth pointing out here that an exact recursive version of the Modified Gram-Schmidt algorithm has recently been proposed by Ling and Proakis[21]. Their algorithm has not yet been investigated for adaptive nulling but has much in common with the QR decomposition technique described below and should perform similarly.

An exponentially fading memory can be incorporated into the sequential decorrelation algorithm by multiplying the terms U(n-1) and V(n-1) in equations (69) and (70) by a simple constant $0 < \beta < 1$. In this more general form, the algorithm requires $\sim 8P^2$ real multiplications and $\sim 6P^2$ real additions per sample time. Since the sequential decorrelation process is computationally more expensive than the Modified Gram-Scmidt technique and the QR decomposition algorithm which is described later, it would appear to be of limited value for digital processing. However it is well suited to analogue implementation since the basic cell function is simply a correlation cancellation loop. The resulting analogue network does not suffer from the fundamental problems of slow convergence associated with closed loop analogue processors.

The sequential decorrelation processor may also be applied to the general adaptive antenna problem by using it in conjunction with a constraint preprocessor of the type described in section 3.1.2.

3.2.4 QR Decomposition

On approach to the least squares estimation problem which is particularly good from the numerical point of view is that of orthogonal triangularization[14] otherwise known as QR decomposition. An n x n orthogonal matrix $\underline{Q}(n)$ is generated such that

$$\underline{Q}(n)\ \underline{X}(n) = \begin{bmatrix} \underline{R}(n) \\ \hline 0 \end{bmatrix} \tag{72}$$

where $\underline{R}(n)$ is a P x P upper triangular matrix. Then, since $\underline{Q}(n)$ is orthogonal we have

$$E(n) = \|\underline{Q}(n)\underline{e}(n)\| = \left\| \begin{bmatrix} \underline{R}(n) \\ \hline 0 \end{bmatrix} \underline{w}(n) + \begin{bmatrix} \underline{u}(n) \\ \hline \underline{v}(n) \end{bmatrix} \right\| \tag{73}$$

where

$$\begin{bmatrix} \underline{u}(n) \\ \hline \underline{v}(n) \end{bmatrix} = \underline{Q}(n)\ \underline{y}(n) \tag{74}$$

It follows that the least-squares weight vector $\underline{w}(n)$ must satisfy the equation

$$\underline{R}(n)\ \underline{w}(n) + \underline{u}(n) = 0 \tag{75}$$

and hence

$$E(n) = \|\underline{v}(n)\| \tag{76}$$

Since the matrix $\underline{R}(n)$ is upper triangular, equation (75) is much easier to solve than the Wiener-Hopf equation (3). The optimum weight vector $\underline{w}(n)$ may be derived quite simply by back-substitution. Equation (75) is also much better conditioned since the condition number of $\underline{R}(n)$ is given by

$$C(R(n)) = C(\underline{Q}(n)\underline{X}(n)) = C(\underline{X}(n)) \tag{77}$$

This property follows directly from the fact that $\underline{Q}(n)$ is orthogonal. $\underline{R}(n)$ is, in fact, the Cholesky square root factor of the covariance matrix $\underline{M}(n)$ although it may be derived as shown here without actually forming the covariance matrix explicitly.

The orthogonal triangularization process may be carried out using either Householder transformations[14] or Givens rotations[15]. The Givens rotation method is particularly suitable for least-squares minimization since it leads to a very efficient algorithm whereby the triangularization process is recursively updated as each new row of data enters the computation. A Givens rotation is an elementary orthogonal transformation of the form

$$\begin{bmatrix} c & s^* \\ -s & c \end{bmatrix} \begin{bmatrix} 0\ldots0, r_i \ldots r_k \ldots \\ 0\cdots0, x_i \ldots x_k \ldots \end{bmatrix} = \begin{bmatrix} 0\ldots0, r_i' \ldots r_k' \ldots \\ 0\ldots0, 0 \cdots x_k' \ldots \end{bmatrix} \quad (78)$$

where $|c|^2 + |s|^2 = 1$ and $c = c^*$. The elements c and s may be regarded as the cosine and since respectively of a (complex) rotation angle θ which is chosen to eliminate the leading element of the lower vector ie such that

$$-sr_i + cx_i = 0 \quad (79)$$

It follows that

$$c = |r_i|/r_i' \text{ and } s = (x_i/r_i).c \quad (80)$$

where

$$r_i' = (|r_i|^2 + |x_i|^2)^{\frac{1}{2}} \quad (81)$$

A sequence of such elimination operations may be used to carry out an orthogonal trangularization of the matrix $\underline{X}(n)$ in a convenient row-recursive manner[16].

The algorithm may be implemented in a very efficient pipelined manner using a triangular systolic array of the type shown in figure 16a. Each cell receives its input data from the directions indicated on one clock cycle, performs the function specified in figure 16b and delivers the appropriate output values to neighbouring cells as indicated on the next clock cycle. Each cell within the basic array (corresponding to its location) stores one element of the recursively evolving triangular matrix $\underline{R}(n)$ which is intialized to zero at the outset of the least squares calculation and then updated every clock cycle. Cells in the right hand column store one element of the evolving vector $\underline{u}(n)$ which is also intialized and updated every clock cycle.

An exponentially fading memory is easily incorporated into the algorithm by multiplying the stored triangular matrix $\underline{R}$ and corresponding vector $\underline{u}$ by a simple constant $0 < \beta < 1$ before implementing each recursive update.

Kung and Gentleman[17] proposed the use of a separate linear systolic array to carry out the back substitution process required to solve equation (75) at the end of the entire recursion. However, for the purposes of deriving the current least-squares residual e(n) it is not necessary to

compute the weight vector $\underline{w}(n)$ explicitly. McWhirter[16] has shown how e(n) may be obtained in a very simple and direct manner which avoids the need to solve equation (75) at any stage of the process and leads to a more reliable and efficient algorithm for recursive least-squares minimization. The current least-squares residual e(n) is simply given by

$$e(n) = \alpha(n) \cdot \gamma(n) \qquad (82)$$

where $\alpha(n)$ is the output which emerges from the bottom cell in the right hand column of the systolic array in Figure 16 and $\gamma(n)$ is the output produced by the lowest boundary cell. The final cell indicated by means of a small circle in Figure 16 simply multiplies the completed product (n) by the other parameter $\alpha(n)$ required to form the least squares residual e(n).

Gentleman has derived a very efficient square root free version of the Givens rotation algorithm[18]. It is not appropriate to describe it here but the corresponding cell functions are given in figure 16c. In this form, the QR decomposition algorithm (including fading memory) requires $\sim 4P^2+24P$ multiplications and $\sim 4P^2+14P$ real additions per sample time.

The adaptive linear combiner illustrated in Figure 16 enjoys all the desirable architectural features of a systolic array. In particular it does not require the block data storage which would be needed in order to implement the modified Gram-Schmidt algorithm. As each row of data moves down through the systolic array it is fully abosrbed into the statistical estimation process, the triangular matrix $\underline{R}(n)$ is updated accordingly and the corresponding residual is produced automatically. The circuit architecture is enhanced by avoiding the need to derive an explicit solution for the least-squares weight vector $\underline{w}(n)$. This leads to a considerable reduction in the amount of computation and circuitry required since it is no longer necessary to clock out each triangular matrix $\underline{R}(n)$, carry out the back substitution or form the output linear combination $\underline{x}^T(n)\underline{w}(n)$. The adaptive linear combiner in Figure 16 is also based on a very stable and well-conditioned numerical algorithm. Indeed the method of QR decomposition by Givens rotations is widely accepted as one of the very best techniques for solving linear least squares problems. However the final triangular linear system may, in general, be ill-conditioned and avoiding the back-substituation process also enhances the numerical properties of the adaptive combiner. In particular the circuit in Figure 16 produces the correct (zero) residual even if $n < p$ and the matrix is not of full rank. This sort of unconditional stability is most important in the design of real time signal processing systems.

The systolic QR decomposition network may of course be applied to the general adaptive antenna problem by using it in conjuction with a constraint pre-processor of the type described in section (3.1.2). Alternatively the constraint may be taken into account by solving a transformed Wiener-Hopf equation analagous to that derived for the modified Gram-Schmidt algorithm in section (3.2.2). The transformed data matrix in this case is obtained by extracting the output residuals for each of the sub array problems, ie for the first P-1 elements, the first P-2

elements and so on. The constraint techniques as applied to the QR method are so similar to those for Gram-Schmidt that no further discussion will be given here.

A systolic array of the type described in this section is currently being developed at the Standard Telecommunications Laboratories in Harlow for use in an adaptive antenna test-bed system.[19] The proposed system was described in some detail during the KTP-3 visit to STL.

3.2.5 The Hung and Turner Algorithm.

We conclude this section by reviewing briefly another novel direct solution algorithm which was recently proposed by Hung and Turner.[20] Their algorithm is designed to be extremely efficient in situations where the number of jamming sources J is known to be very much less than the number of weighting coefficients P. The first J data snapshots (ie the first J rows of the data matrix $\underline{X}$) are assumed to span the space defined by the J jammer response vectors and an orthonormal basis set for the space is generated by carrying out a Gram-Schmidt orthogonalization of the J snapshot vectors. The adaptive weight vector $\underline{w}$ is then obtained by subtracting from the quiescent weight vector $\underline{w}_q$ its projected component on each of these basis vectors. The adapted weight vector should then be orthogonal to each of the J jammer signals as required.

In the Hung and Turner algorithm, the orthogonalization procedure is carried out on the first J rows of the data matrix and not on the entire P columns as with the modified Gram-Schmidt technique described in section (3.2.2). As a result, the total number of arithmetic operations required is $\sim 8JP$ which is much less than than required for the modified Gram-Schmidt algorithm when $J \ll P$.

Hung and Turner have shown their method to be effective in the suppression of many types of jammer sources and in the case of monotone point sources the output jammer power is reduced to a few dB above the white noise background. Their method is bound to be numerically less stable and accurate than the modified Gram-Schmidt or QR algorithms but it appears none the less to be very cost effective in the appropriate circumstances.

4 EFFECTS OF MULTIPATH

In this section we will consider separately the effect of multipath propogation on the jammer signals and on the desired signal.

4.1 Jammer Multipath.

Consider first a multipath jammer return which does not enter the main beam or desired look direction of the array. If it is uncorrelated this return appears as an additional jammer signal and simply absorbs another of the available degrees of freedom. If, on the other hand, the multipath return is fully correlated, the array can cancel both the jammer and its multipath by steering a single null in the appropriate vector balanced direction.

Consider next the effect of a multipath jammer signal which enters the array through the main beam or required look direction. If the multipath return is uncorrelated it cannot be cancelled by adaptive nulling. If the return is correlated then it can at least be partially cancelled by the adaptive algorithm. In the case of SLC the cancellation will be greatly limited since the gain of the main antenna is assumed to be much greater than that of the auxiliaries. For a general antenna array with a look direction constraint the multipath signal can in principle be cancelled completely depending on the gain assigned to the look direction.

In summary then, correlated multipath jammer returns cause less of a problem than uncorrelated ones since they may be cancelled to some extent without increasing the number of degrees of freedom in the system.

4.2 Signal Multipath.

The situation is reversed for signal multipath in which case correlated returns cause a much greater problem that uncorrelated ones. Consider a signal multipath return which enters the array along a path distinct from the mian beam or required look direction. If this return is uncorrelated it cannot cause any degradation of the desired signal and may be nulled independently. It will simply absorb an additional degree of freedom in the process. If the multipath signal is correlated, however, the effect can be very damaging. In principle the multipath return could cancel the desired signal completely. In the case of SLC the degradation will be limited due to the high gain of the primary antenna. However, with a general antenna array the degradation may be much more serious depending on the gain which is chosen for the look direction. The only solution to this problem is to detect and elminate multipath signal returns before carrying out any adaptive nulling. However, since the signal multipath returns are often separated from the direct path by a very small angle, detecting and eliminating them independently requires a high degree of spatial resolution. This is one of the reasons why enhanced resolution algorithms are considered to be important and are discussed in the next section of this report.

REFERENCES

1 Applebaum S P, Adaptive Arrays, IEEE Trans Ant and Prop, vol AP-24, p 585 (1976).

2 Dokter R A, Masenten W K and Kinkel J F, "Trends in Adaptive Antenna Circuit Design", Proc ELECTRO 79 (Feb 1979).

3 Widrow B, Mantey P E, Griffiths L J and Goode B B, "Adaptive Antenna Systems", Proc IEEE, vol 55, p 2143 (1967).

4 Frost O L, "An Algorithm for Linearly Constrained Adaptive Array Processing", Proc IEEE, vol 60, p 926 (1972).

5 Griffiths L J, "A Simple Adaptive Algorithm for Real-Time Processing in Antenna Arrays", Proc IEEE, vol 57, p 1696 (1969).

6 McWhirter J G, "Systolic Array for Recursive Least Squares Minimization", Electronics Letters, vol 19, No 18, p 729 (Sept 1983).

7 Masenten W K, "Adaptive Signal Processing", IEE Seminar "Case Studies in Advanced Signal Processing", Peebles, Scotland (Sept 1979).

8 Reed I S, Mallett J D and Brennan L E, "Rapid Convergence Rate in Adaptive Arrays", IEEE Trans Aerosp Electron Syst, vol AES-10, p 853 (1974).

9 "Radar Signal Processing Architectures and Algorithms for VLSI", TTCP Technical Panel KTP3, Technical Report TR-5.

10 "Adaptive Array Principles" by J E Hudson, Peter Peregrinus (1981).

11 "Dillard G M, "Processing Requirements for the Gram-Schmidt Procedure Applied to a Digital Adaptive Array", TTCP Technical Panel KTP-3, Technical Memo TM-T (June 1981).

12 Bjork A, "Solving Linear Least Squares Problems by Gram-Schmidt Orthogonalization", BIT, p 1 (1967).

13 "Introduction to Adaptive Arrays", by Monzingo R A and Miller T W, Wiley Interscience (1980).

14 Golub G, "Numerical Methods for Solving Linear Least Squares Problems", Numerische Mathematik, vol 7, p 206 (1965).

15 Givens W, "Computation of Plane Unitary Rotations Transforming a General Matrix to Triangular Form", J Soc Indust Appl Math, vol 6, No 1, p 26 (1958).

16 McWhirter J G, "Recursive Least Squares Minimization using a Systolic Array", Proc SPIE, vol 431, "Real Time Signal Processing VI" (1983).

17 Kung H T and Gentleman W M, "Matrix Triangularization by Systolic Arrays", Proc SPIE, vol 298, Real Time Signal Processing IV" (1981).

18 Gentleman W M, "Least Squares Computations by Givens Rotations Without Square Roots", J Inst Maths Applics, vol 12, p 329 (1973).

19 Ward C R, Robson A J, Hargrave P J and McWhirter J G, "Application of a Systolic Array to Adaptive Beamforming", Proc IEE, vol 131, Pt F, No 6 (1984).

20 Hung E K L and Turner R M, "An Adaptive Jammer Suppression Algorithm for Large Arrays", McMaster University (1981).

21 Ling F and Proakis J G, "A Recursive Modified Gram-Schmidt Algorithm with Application to Least Squares Estimation and Adaptive Filtering", Proc IEEE conf ISCAS '84.

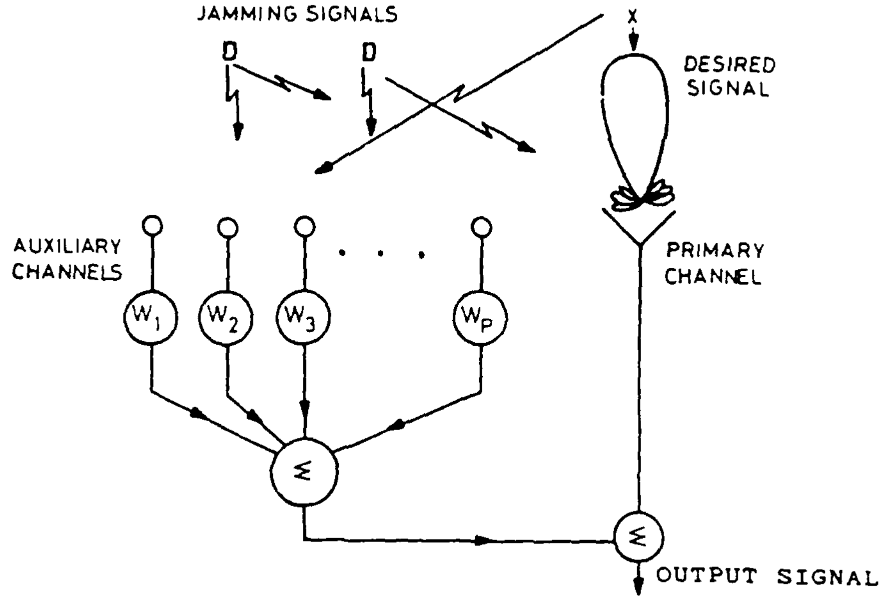


Figure 1. Adaptive side-lobe canceller

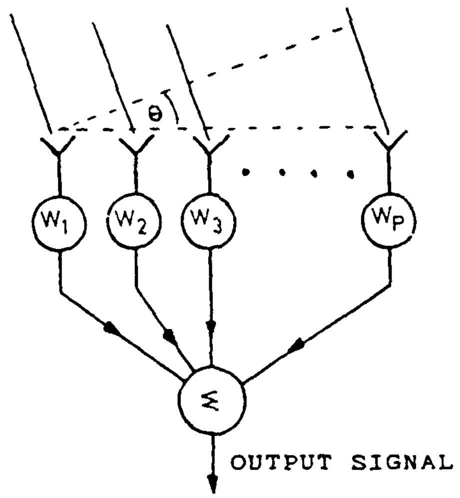


Figure 2. General adaptive antenna array

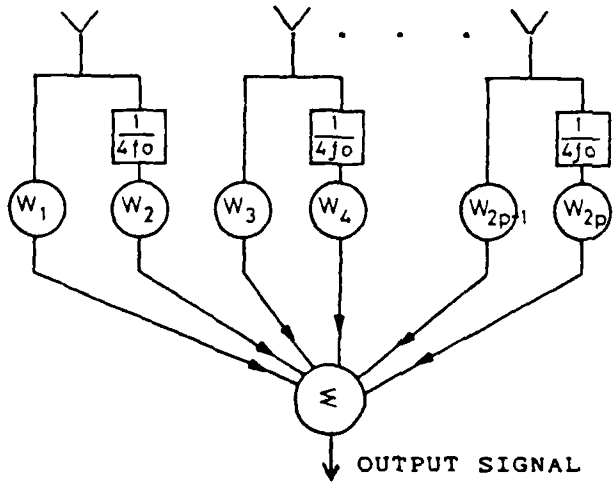


Figure 3. Adaptive combiner using real inputs

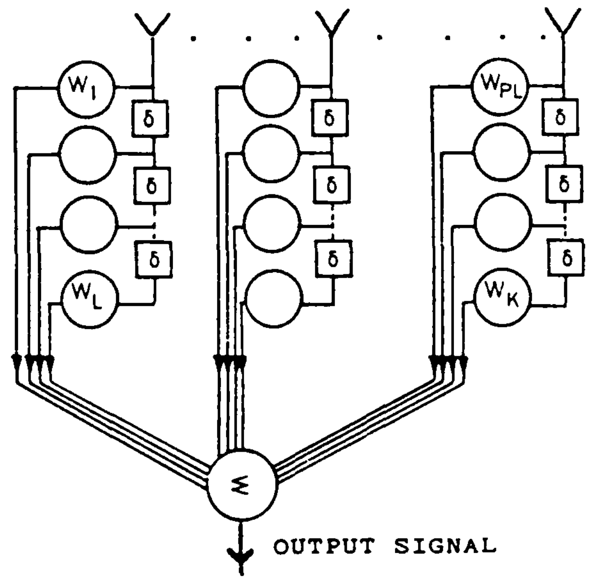


Figure 4. Broad band adaptive antenna

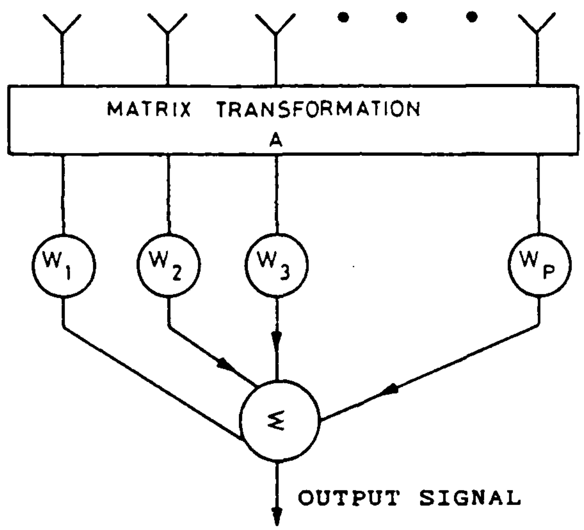


Figure 5. "Beam Space" adaptive combiner

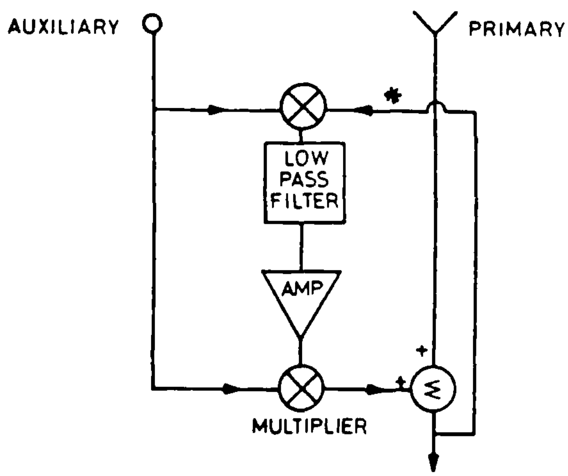


Figure 6. Analogue correlation cancellation loop

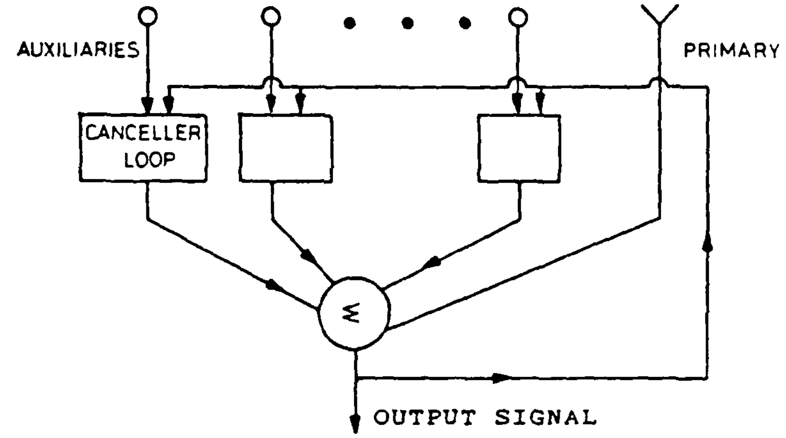


Figure 7. Use of multiple cancellation loops for SLC

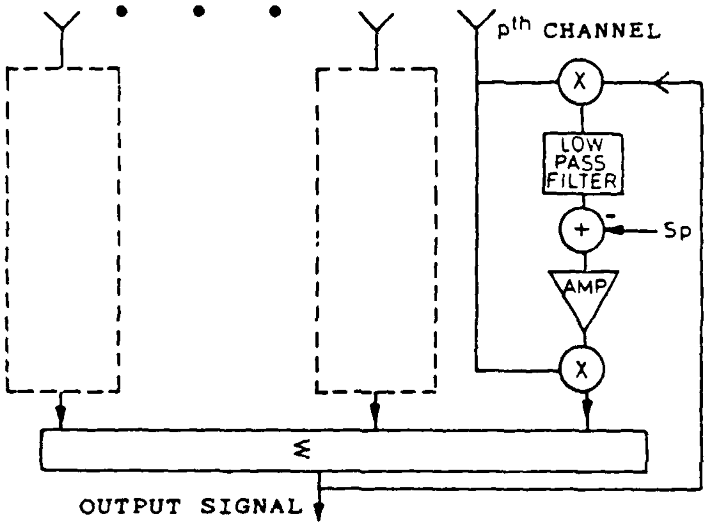


Figure 8. Use of multiple cancellation loops for general array

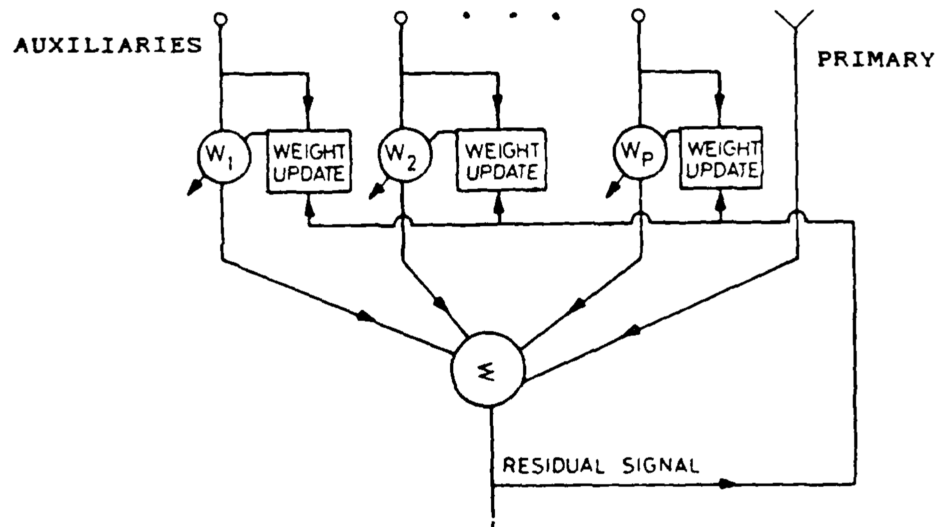


Figure 9. LMS algorithm applied to SLC

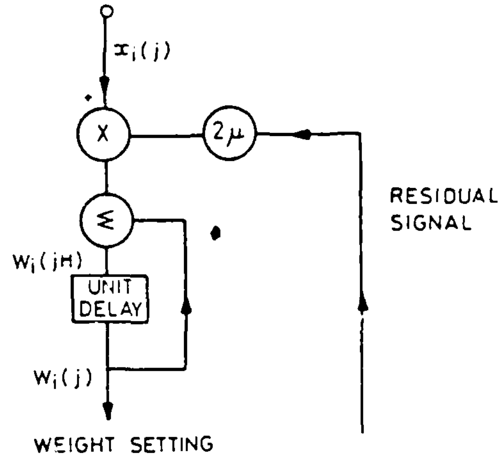


Figure 10. Digital correlation cancellation loop

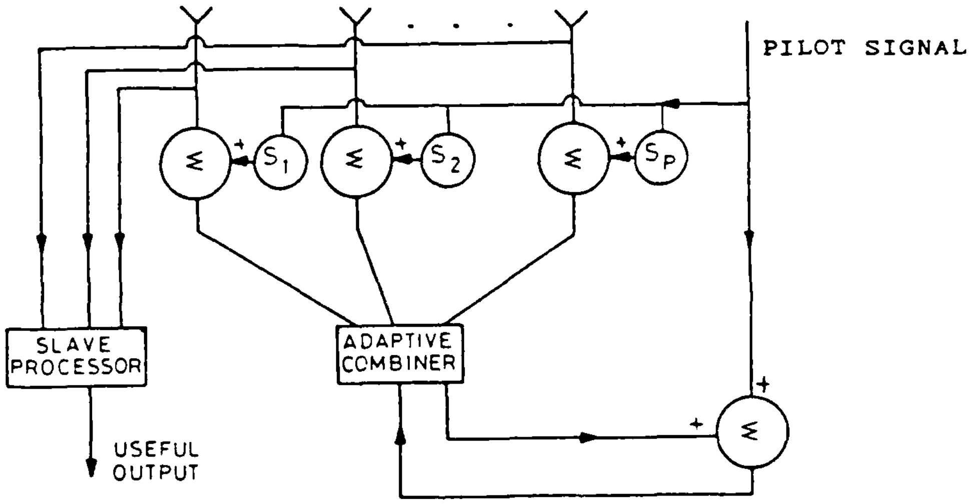


Figure 11. LMS algorithm applied to general array

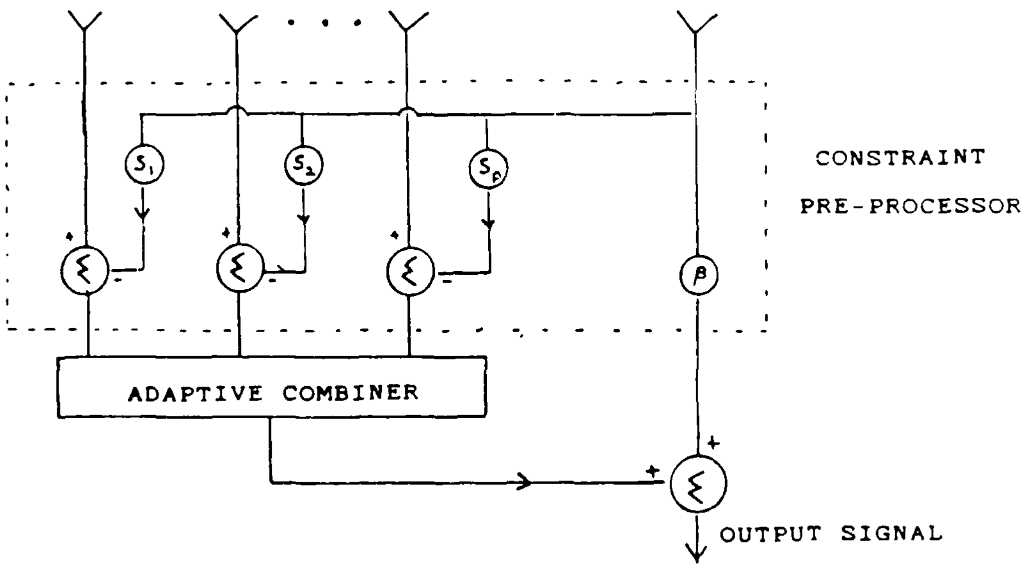


Figure 12. Linear constraint pre-processor

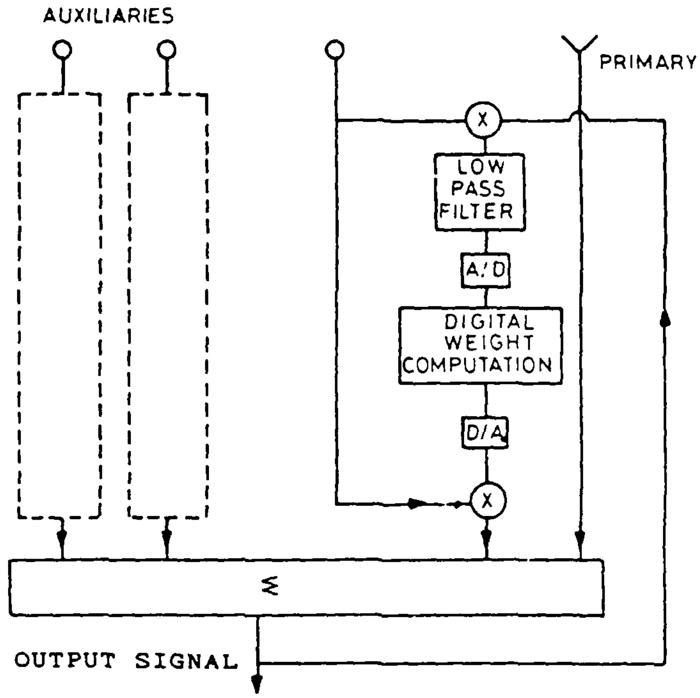


Figure 13. Use of hybrid analogue/digital loops for SLC

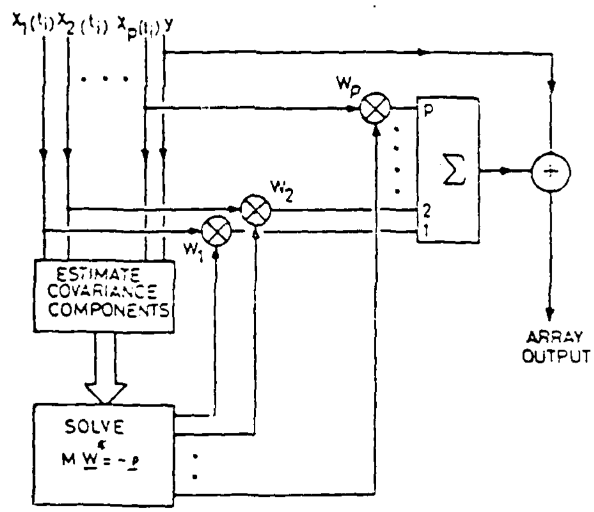


Figure 14. SMI processor architecture

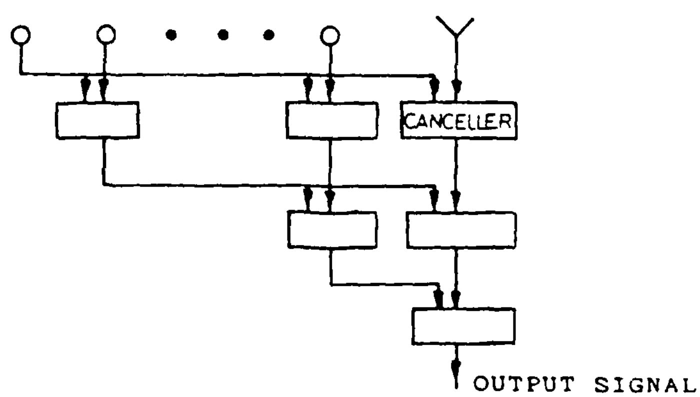


Figure 15. Gram-Schmidt/Sequential decorrelation architecture

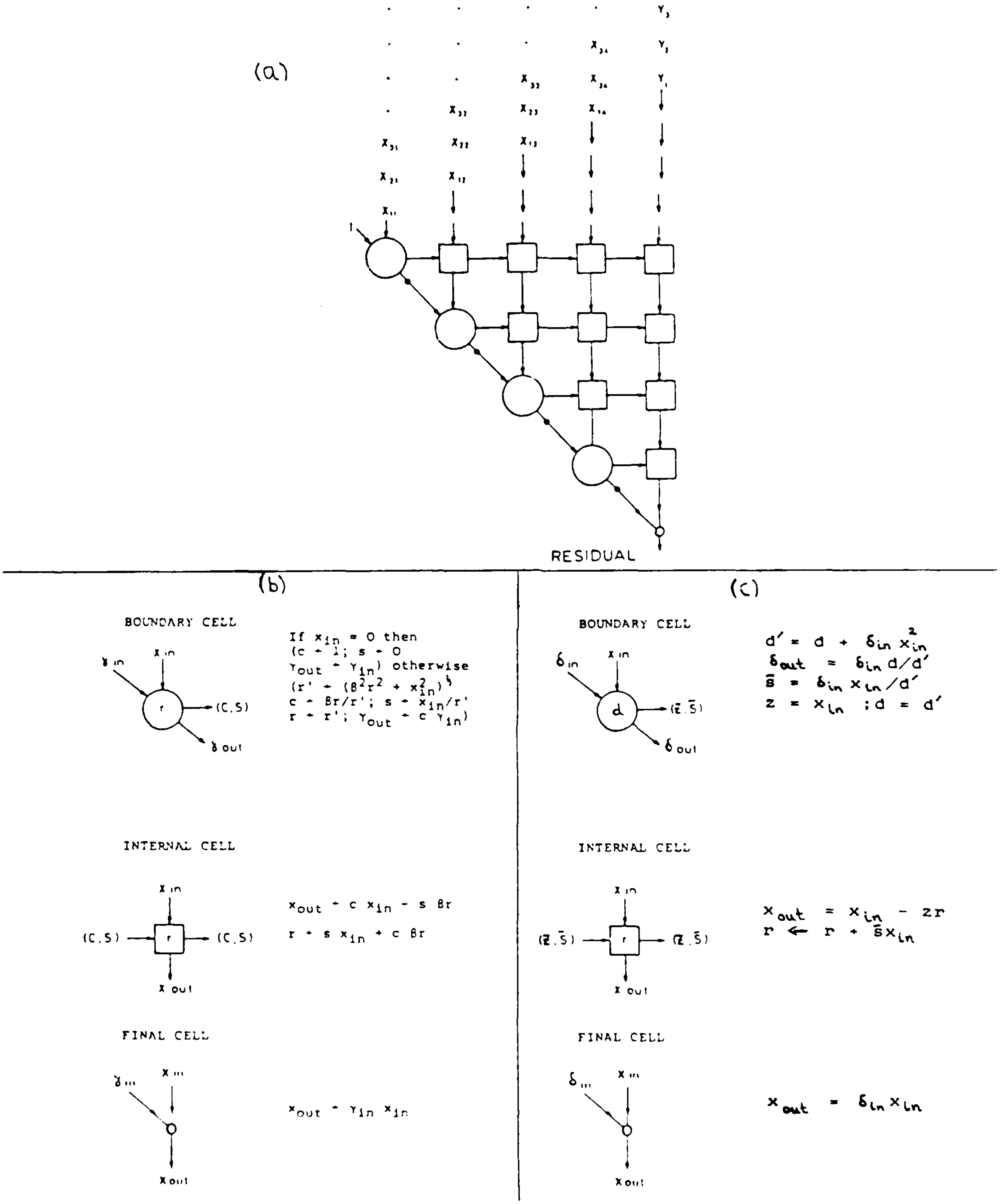


Figure 16. (a) Systolic array for QR decomposition
(b) Cells required for basic Givens rotation method
(c) Cells required for square root free algorithm

DOCUMENT CONTROL SHEET

Overall security classification of sheet ....Unclassified........

(As far as possible this sheet should contain only unclassified information. If it is necessary to enter classified information, the box concerned must be marked to indicate the classification eg (R) (C) or (S))

1. DRIC Reference (if known)	2. Originator's Reference Memorandum 3939	3. Agency Reference	4. Report Security Classification U/C
5. Originator's Code (if known)	6. Originator (Corporate Author) Name and Location Royal Signals and Radar Establishement		
5a. Sponsoring Agency's Code (if known)	6a. Sponsoring Agency (Contract Authority) Name and Location		

7. Title
A Brief Review of Adaptive Null Steering Techniques
7a. Title in Foreign Language (in the case of translations)
7b. Presented at (for conference papers) Title, place and date of conference

8. Author 1 Surname, initials	9(a) Author 2	9(b) Authors 3,4...	10. Date	pp. ref.
McWhirter J G				

11. Contract Number	12. Period	13. Project	14. Other Reference

15. Distribution statement

Unlimited

Descriptors (or keywords)

continue on separate piece of paper

Abstract A brief theoretical review of adaptive null steering is presented. The basic theory is first outlined in the context of sidelobe cancellation systems as well as general antenna arrays. Various approaches to the practical implementation of adaptive null steering are then disucssed. These fall into the two main categories of closed loop mehtods and direct solution methods. The closed loop methods are very cost-effective and suitable in principal, for either analogue or digital processing. However their rate of convergence is fundamentally limited and too slow for some applications. The direct solution methods do not suffer from this problem but tend to be suitable only for digital processing and are more expensive from the computational point of view. However they are well suited to parallel processing and now provide a very practical alternative due to recent advances in VLSI cirucit technology. A brief discussion on the effects of multi-path propagation on adaptive null steering systems concludes this brief review.

S80/48

END
DTIC
8-86